

Lisbon Young Mathematicians Conference

April 11 & 12 · Lisbon, Portugal

Program & Book of Abstracts



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FOREWORD

We warmly welcome all participants to the Lisbon Young Mathematicians Conference 2025 (LYMC 2025), held on 11th and 12th April 2025, at Instituto Superior de Economia e Gestão da Universidade de Lisboa (ISEG – Lisbon School of Economics and Management, University of Lisbon), within the framework of the Department of Mathematics and Centro de Matemática Aplicada à Previsão e à Decisão Económica (CEMAPRE).

LYMC 2025 is the fifth edition of a project conceived by a group of five senior researchers from five university institutions at the Lisbon region, who have since set up the LYMC Organising and Scientific Committe. Their initiative and determination were unwavering to launch this series of annual conferences aimed at bringing together postgraduate students and early career researchers in mathematics, statistics and their applications.

The first edition was held in 2021 at NOVA School of Science and Technology, where the event was held entirely in virtual mode due to the COVID-19 pandemic. The next editions were held at Faculty of Sciences/CIÊNCIAS ULisboa, Universidade Aberta, and Instituto Superior Técnico.

For every edition, four distinguished researchers were invited. It is with great honour that we welcome our distinguished plenary speakers of LYMC 2025, who will enrich the conference with their participation and be an inspiration to our audience:

Hansjoerg Albrecher, Faculté des HEC, Université de Lausanne,

Benedetta Ferrario, Università di Pavia,

James Kennedy, Faculdade de Ciências, Universidade de Lisboa,

Paulo Mateus, Instituto Superior Técnico, Universidade de Lisboa.

The ISEG Organising Committee received and approved 33 oral presentations. The abstracts of these presentations, as well as those of our distinguished plenary speakers, can be found in this booklet.

We would like to thank all the authors for their contributions and the organizing institutions and research centers for their support. A special word of recognition is due to the LYMC Organising and Scientific Committee, who have always been committed to each of the five editions, articulating with the respective Local Organising Committee.

Thanks are also due to Inês Lami, Science Manager of CEMAPRE, Susana Branco, Assessor of the Department of Mathematics, as well as to the ISEG Marketing & External Relations team, namely, Paula Monteiro, Alice Vieira and Joel Joaquim, for their collaboration in the organisation of this event.

After five editions, we believe the LYMC project has proved successful. We are hopeful that a new round will begin in 2026, building on and enhancing the valuable experiences gained.

We wish you an inspiring and enriching experience at LYMC 2025!

Thank you all!

The Local Organizers

Maria do Rosário Grossinho João Guerra Manuel Guerra João Janela

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PROGRAM

All lecture rooms on the 2nd floor, Building Quelhas 6. Room 202 (same floor) is available for free discussions or group meetings.

Friday, April 11th

9:00	Registration – Auditorium CGD		
9:30	Opening Session – Auditorium CGD		
10:00	Plenary Talk I – Auditorium CGD(Chair: Carlota Rebelo)James Kennedy: Drums that (sometimes) sound the same		
10:45	Coffee break		
11:15	Parallel Sessions I		
	Session IA – Auditorium CGD (Chair: Pedro Serranho)	Session IB – Auditorium 2 (Chair: Manuel Guerra)	
	João Afonso: Analysis of Dirichlet bound- ary control of the Stokes equation with mixed boundary conditions	Carlos Carteiro: <i>Maximal functions of</i> <i>Toeplitz kernels</i>	
	Muhammad Adnan Anwar: <i>Time-</i> <i>dependent strategy for improving aortic</i> <i>blood flow simulations with boundary control</i> <i>and data assimilation</i>	Alina Shalukhina: Maximal operator on variable Lebesgue spaces: Euclidean vs. spaces of homogeneous type	
	Simão Eusébio: Specialized finite difference method for linear partial differential equa- tions	Sandra Thampi: <i>The Brown-Halmos theo-</i> <i>rem for discrete Wiener-Hopf operators</i>	
	Razi Khan: Numerical analysis of shear- dependent unsteady non-Newtonian fluid flow and heat transfer in open-ended cavities	Márcio Valente: Fredholm criteria for sin- gular integral operators with continuous co- efficients on variable Lebesgue spaces	
	Yashveer Kumar: Numerical approxima- tion scheme based on computational method to solve distributed order fractional financial mathematical model	Ghulam Abbas Khan: Spline collocation method for a class of singular fractional differential equations of an order $\beta \in (0, 1)$	
12:30	Lunch		
14:30	Plenary Talk II – Auditorium CGD (Chair: João Janela) Paulo Mateus: Open problems in quantum computation and cryptography: A mathematical perspective		
15:15	Coffee break		

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15:45	Parallel Sessions II		
	Session IIA – Auditorium CGD (Chair: Raquel Bernardino)	Session IIB – Auditorium 2 (Chair: João Araújo)	
	Francisco Canas: Formulations and branch- and-cut algorithms for cycle covers with up to p cycles	Diogo Freire de Andrade: The bordism $(\infty, 1)$ -category as a Morita category of stratified spaces	
	Sofia Henriques: Formulations and branch- and-cut algorithms for the period travelling salesman problem	Guilherme Azevedo: Partition regularity of Pythagorean triples	
	Tomás Kapancioglu: Integer linear pro- gramming models for the travelling thief problem	Maria Osório: <i>Bilateral base-extension se-</i> <i>mantics</i>	
	Parisa Ahani: <i>Optimizing decision-making framework under uncertainty for transport operations</i>	Tânia Paulista: Classes of semigroups and their commuting graphs	
	Jacopo Strada: <i>Physico-chemical bound-</i> <i>ary conditions for multiphysics modeling of</i> <i>rechargeable Zn-Air flow batteries</i>	Ana Catarina Monteiro: <i>Product of forma-</i> <i>tions and Fitting classes on groups and some</i> <i>generalisations</i>	
	Maria Grazia Quarta: A deep learning approach to parameter estimation in battery modeling for charge-discharge processes	Faustino Maciala: On the Drazin inverse of double star digraph matrices: Index characterization and pseudo-inverses	

Saturday, April 12th

9:00	Registration – Auditorium CGD		
9:15	Plenary Talk III – Auditorium CGD(Chair: Fernanda Cipriano)Benedetta Ferrario: Stochastic fluid dynamics		
10:15	Parallel Sessions III		
	Session IIIA – Auditorium CGD (Chair: Maria do Rosário Oliveira)	Session IIIB – Auditorium 3 (Chair: Fernanda Cipriano)	
	Vítor Augusto: <i>PSInference: A package to draw inference for released Plug-in Sampling single synthetic dataset</i>	Kush Kinra: Global-in-time optimal control of stochastic third-grade fluids	
	Felipe Barletta: Bayesian joint analysis to severity of illness and length of stay of pa- tients using Portuguese hospital morbidity database	Raya Nouira: Continuous data assimilation for a class of third-grade fluids equations in 2d and 3d	
	Vera Pinto: <i>Statistical considerations in variant calling: A performance-based comparative study</i>	Anita Ribeiro: <i>Shape optimization problems</i> <i>in PDE, an introduction</i>	
	João F. Carrilho: <i>Topic analysis on glioma</i> <i>transcriptomics for improved disease charac</i> - <i>terization and biomarker discovery</i>	Diogo Franquinho: Neural network empow- ered liquidity pricing in a two-price economy under conic finance settings	
	Luís Garcez: tradeoffaucdim: An R pack- age for accessing the trade-off between model performance and dimensionality	João Paixão: A hybrid method for the inverse transmission problem of scattering a time-harmonic acoustic wave by a penetrable object	
	João Rodrigues: <i>Proximity measures and outlier detection for mixed-type data</i>		
11:45	Coffee break		
12:15	Plenary Talk IV – Auditorium CGD (Chair: Manuel Guerra) Hansjörg Albrecher: Mathematics for insurance, the modeling of NatCat risks and change		

13:00 Closing Session – Auditorium CGD

ABSTRACTS

PLENARY SPEAKERS

Mathematics for insurance, the modeling of NatCat risks and climate change

Hansjörg Albrecher^{1,*}

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Abstract

In this talk it will be highlighted how various techniques from mathematics can be used to address questions in insurance, including probability, complex analysis, integro-differential equations and techniques from algebra and symbolic computation. More detail will then be given on techniques to model NatCat risks and a discussion of the specific challenges and needs for their insurance in the light of climate change.

Stochastic fluid dynamics

Benedetta Ferrario^{1,*}

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Abstract

I will present some mathematical problems of stochastic fluid dynamics, that have caught my interest along the years. I will consider the basic equations modeling the motion of incompressible viscous fluids, the Navier-Stokes equations, in which the driving force has a random component, the so-called white noise. For these equations I will present some of the main results concerning the statistical description of the motion and some open problems, related to the mathematical analysis of turbulence. In particular I will review different sufficient conditions on the noise forcing term which grant the existence of a unique stationary probability distribution and its role in describing the asymptotic behavior of the fluid motion.

Drums that (sometimes) sound the same

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Abstract

A common family of inverse problems in mathematical physics involves extracting information about a domain or manifold from properties of the spectrum of a differential operator such as a Laplacian defined on it. Since the eigenvalues of the Laplacian correspond to the resonant frequencies of a vibrating object in the shape of the domain, this question is sometimes reformulated as "what properties of a drum can one hear?". A much stronger form of this question, famously posed by Marc Kac in the 1960s and answered in the negative in the 1990s, is, does the spectrum of the Laplacian uniquely determine a domain up to congruence, that is, "can one hear the shape of a drum?". We will give a brief introduction to this problem, including what evidence Kac had when he formulated his question, and examples of "different drums that sound the same". We will then sketch an elementary proof (adapted from some of the 1990s ones) of the isospectrality of the corresponding domains, and finish by showing how slight changes to the form of the operators involved in the question can lead to a complete breakdown of the isospectrality result and to deep open problems. Parts of this talk are based on joint work with Wolfgang Arendt and A.F.M. ter Elst.

Open problems in quantum computation and cryptography: A mathematical perspective

Paulo Mateus^{1,*}

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Abstract

In this talk, we will revisit key open questions in quantum computation and cryptography from a mathematical perspective. We'll start by examining the potential falsification of the extended/efficient Church-Turing thesis due to Shor's algorithm and its implications for complexity theory. We'll then explore the impact of quantum computation on modern cryptography, highlighting the main approaches to address these challenges, specifically quantum and post-quantum cryptography. Finally, we'll conclude with a discussion of significant open questions that remain in the field.

CONTRIBUTED TALKS

Analysis of Dirichlet boundary control of the Stokes equation with mixed boundary conditions

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Abstract

We study Dirichlet boundary control of the Stokes equation for incompressible fluids in the unit square domain with mixed boundary conditions. We follow previous studies of L^2 -regularization through the standard velocity tracking cost functional as seen in [1] and [2]. The main goal of this study is to introduce non-homogeneous Neumann boundary conditions and analyse the regularity and convergence of the solution of the boundary control problem in a mixed boundary setting. To that end, we adapt some proofs of well-posedness, regularity results and optimality conditions from [1] and [2], as well as a finite element discretization from [1] and [3]. We develop a Taylor-Hood finite element method based algorithm to solve the optimization problem in this mixed boundary setting and derive a priori error estimates for it. Some numerical experiments are presented.

Keywords: Dirichlet boundary control, mixed boundary conditions, Neumann boundary conditions, Stokes equation, finite element method, regularity.

References

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Optimizing decision-making framework under uncertainty for transport operations

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Abstract

This research addresses the fleet composition problem for a transport operator over a defined planning period, considering uncertainties in energy costs, vehicle prices, and operational expenses. The decision-maker, operating under a limited budget, must strategically structure the fleet by selecting from various vehicle types, each with distinct cost and performance characteristics. For instance, diesel vehicles typically have a lower initial purchase cost but higher operational and maintenance expenses, whereas electric vehicles incur higher upfront costs but benefit from lower running costs [1], [2]. Given the volatility of energy prices, particularly oil, incorporating uncertainty into fleet

planning is critical to avoiding excessive long-term costs. Relying solely on conventionally fueled vehicles due to their lower purchase cost could lead to higher total cost of ownership over time. To mitigate such risks, this study proposes a novel multi-objective mixed-integer quadratic programming model aimed at minimizing both total cost, which includes purchase, energy, maintenance, depreciation, and emission costs, and the financial risk associated with uncertain parameters such as energy prices and vehicle costs. The model generates a Pareto front, enabling decision makers to evaluate trade offs between total cost and risk, providing a data driven framework for cost effective and resilient fleet management strategies [3].

Keywords: Multi-Objective Optimization, Epsilon-constraint method, Sustainable freight fleet, Risk analysis

References

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The bordism $(\infty, 1)$ -category as a Morita category of stratified spaces

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Abstract

In this talk, I will present joint work in progress with David Ayala and John Francis on constructing the bordism $(\infty, 1)$ -category as a Morita category associated to a bespoke category of stratified spaces—first constructed in [3][1][4]. Our approach employs the theory of exponentiable fibrations and relative functor categories, developed in [2], in order to gel the structures of classical geometric bordisms within a rigorous higher-categorical framework. I will explain how this shift in perspective clarifies and enriches our understanding of bordism categories, providing a suitable foundation for constructing topological quantum field theories using β -factorization homology.

Keywords: Bordism categories, Morita categories, Stratified spaces, Factorization Homology.

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- [2] Ayala D., Francis J.G. "Fibrations of ∞-categories." Higher Structures, Vol. 4, No. 1, pp. 168-265, 2020.
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Time-dependent strategy for improving aortic blood flow simulations with boundary control and data assimilation

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Abstract

Understanding time-dependent blood flow dynamics in arteries is crucial for diagnosing and treating cardiovascular diseases. However, accurately predicting time-varying flow patterns requires integrating observational data with computational models in a dynamic environment. This study investigates the application of data assimilation and boundary optimization techniques to improve the accuracy of time-dependent blood flow simulations. We propose an integrated approach that combines data assimilation methods with boundary optimization strategies tailored for time-dependent cases. Our method aims to minimize the disparity between model predictions and observed data over time, thereby enhancing the fidelity of time-dependent blood flow simulations. Using synthetic timeseries observational data with added noise, we validate our approach by comparing its predictions with the known exact solution, computing the -norm to demonstrate improved accuracy in timedependent blood flow simulations. Our results indicate that the optimization process consistently aligns the optimized data with the exact data. In particular, velocity magnitudes showed reduced discrepancies compared to the noisy data, aligning more closely with the exact solutions. The analysis of pressure data revealed a remarkable correspondence between the optimized and exact pressure values, highlighting the potential of this methodology for accurate pressure estimation without any previous knowledge on this quantity. Furthermore, wall shear stress (WSS) analysis demonstrated the effectiveness of our optimization scheme in reducing noise and improving prediction of a relevant indicator determined at the postprocessing level. These findings suggest that our approach can significantly enhance the accuracy of blood flow simulations, ultimately contributing to better diagnostic and therapeutic strategies.

Keywords: Adjoint problem, Data assimilation, Boundary control.

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PSInference: A package to draw inference for released Plug-in Sampling single synthetic dataset

Vítor Augusto^{1,*}, Ricardo Moura^{1,2}, Mina Norouzirad¹, Miguel Fonseca^{1,3}

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Abstract

The development and dissemination of synthetic data play a crucial role in statistical disclosure control, balancing data utility and privacy. In this presentation, we are introducing PSInference [1], an R package designed to perform exact inferential analysis on singly imputed synthetic data generated through Plug-in Sampling, assuming an underlying multivariate normal distribution. Unlike traditional methods that rely on multiple synthetic datasets, PSInference enables robust statistical inference from a single synthetic dataset, addressing practical constraints such as computational limitations and data privacy policies. The package includes functions for testing covariance structures, sphericity, independence between variable subsets, and regression analysis. Numerical studies using Monte Carlo simulations confirm the accuracy and reliability of the package's inferential methods.

We also present the underlying methodology to extend this work to the Bayesian realm, with the purpose of establishing a threshold of comparison for the usual non-parametric techniques of creating synthetic data used in the literature.

Keywords: Plug-in Sampling, Synthetic Data, PSInference.

References

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Partition regularity of Pythagorean triples

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Abstract

A central question in Ramsey theory is concerned with determining the patterns that must appear in a single cell for every partition of the set of natural numbers $\mathbb{N} = \{1, 2, ...\}$ into finitely many cells. Equations satisfying the property that, given any finite partition of \mathbb{N} , there is a solution with all the variables belonging to the same cell are called *partition regular*. We will focus on the problem of determining the partition regularity of the Pythagorean equation $x^2 + y^2 = z^2$, colloquially referred to as the *partition regularity problem for Pythagorean triples*. This question was first posed in the late 70's, and until the last decade there were few developments on this topic. In 2016, the case where \mathbb{N} is partitioned into two sets was verified with the help of a computer search.

If we restrict ourselves to partitions given by multiplicative functions (functions $f : \mathbb{N} \to \mathbb{C}$ such that f(xy) = f(x)f(y)) taking finitely many values, then the answer to the Pythagorean triples problem is affirmative [1]. In this talk, we give a brief overview of how this is achieved and how it can be extended to finite colourings induced by arbitrary multiplicative functions.

Keywords: Partition Regularity, Pythagorean Triples, Multiplicative Functions.

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Bayesian joint analysis to severity of illness and length of stay of patients using Portuguese hospital morbidity database

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Abstract

Heart failure (HF) is one of the leading causes of hospitalization and mortality in Portugal, with a rising prevalence projected to increase by 30% by 2035 comparatively to 2011 [1]. This hospitalization study is based on Hospital Morbidity Database (BDMH), covering public hospital admissions in mainland Portugal between 2010 and 2018. The focus is analyze the impact of HF severity to length of stay (LOS), considering hospital heterogeneity to taking into account the in-hospital mortality risk. A Bayesian joint modeling approach was employed using with integrated nested Laplace approximation (INLA) [2] The proposed joint model can evolves multiple submodels [3] integrating a generalized linear mixed model (binomial model) to severity of illness with random hospital effects) and a survival model (Weibull and random walk baseline functions) to predict the LOS. The proposed joint model revealed the shared random effects structure improved risk estimation and enhances survival predictions. In general, the hazard function trajectories predicted by the joint model were lower compared those ones based on model without shared terms, suggesting that neglecting hospital-observed heterogeneity may lead to overestimated hazard function projections. The findings highlight the advantages of joint modeling for HF hospitalization, providing better and individualized hazard function predictions.

Keywords: Hospitalization length of stay, Heart failure severity, Joint modeling, Bayesian analysis.

References

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Formulations and branch-and-cut algorithms for cycle covers with up to p cycles

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Abstract

Given a positive integer p and a weighted undirected graph G = (V, E), we study a problem in which the objective is to find a minimum weight set of up to p elementary cycles partitioning the vertices of G. We study several exponential sized formulations including i) edge variables only; ii) edge and depot variables only; iii) edge, depot and node-depot assignment (NDA) variables only; iv) edge, depot, NDA and edge-depot assignment (EDA) variables, some of which are known from existing works on similar problems (see, e.g., [1], [2], [3]). Relations between all the formulations are established. Branch-and-cut algorithms based on many of these formulations are proposed, and computational experiments are conducted to compare the performance of the different algorithms. The computational testing reveals that some of the formulations including edge, depot and NDA or EDA variables produce the best initial lower bounds and that the best computational times are obtained with the algorithms based on formulations including edge and depot variables only. The best performing algorithm (in terms of computational times) is capable of solving several instances with up to 442 nodes for different values of p.

Keywords: Combinatorial optimization, Integer linear programming, Location-routing, Branch-and-cut.

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Topic analysis on glioma transcriptomics for improved disease characterization and biomarker discovery

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Abstract

Gliomas are highly heterogenous brain tumors with generally poor prognoses, making the search for novel biomarkers and targeted therapies crucial. We analyzed glioma RNA-seq data from The Cancer Genome Atlas first with latent Dirichlet allocation [1] to identify latent gene expression patterns (topics) across glioma subtypes. We then assessed the predictive performance of the most expressed genes (top genes) using linear discriminant analysis, comparing it to the sparse version of this method on the full dataset. The graphical lasso algorithm [2] enabled the exploration of the gene network structure and related it to the inferred topic distributions. Finally, we performed differential expression analysis [3] integrating STRING core data to highlight functional associations.

Our findings reveal distinct transcriptional signatures across glioma subtypes. In particular, the genes GFAP and CLU show consistently high proportions in several topic distributions and strong correlation in the graphical lasso. The top genes achieved an accuracy of 86% in discriminant analysis, comparable to a group of variables selected for classification purposes while retaining relevant information regarding topic structure. We also observed strong topic-specific subnetworks within the full network with graphical lasso. Differential expression performed in both the top genes and the ones associated to those according to STRING revealed sets of genes that may play an important role in distinguishing between glioblastoma and lower grade glioma, as NEFM and VIM, respectively.

Keywords: Gliomas, topic analysis, classification, network analysis, differential analysis.

References

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Maximal functions of Toeplitz kernels

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Abstract

A Toeplitz operator is a compression of a multiplication operator on the Hardy space $H^2(\mathbb{D})$, and we are interested in studying the kernels of such operators, known as Toeplitz kernels.

Every nonzero function f in $H^2(\mathbb{D})$ belongs to a minimal Toeplitz kernel $K_{min}(f)$, meaning that if f is in another Toeplitz kernel K, then $K_{min}(f) \subset K$. On the other hand, a Toeplitz kernel can have multiple maximal functions, i.e., it can be the minimal kernel of several linearly independent functions.

In this talk, we examine results on the maximal functions that can be found within a given Toeplitz kernel.

Keywords: Toeplitz operators, maximal functions.

Specialized finite difference method for linear partial differential equations

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Abstract

The finite difference method is a very well-known and well-established method of approximation of the value of a function u at a point x, u(x), using its values at other ones, $\{u(y_i)\}_i$, in a neighbourhood of the first. This involves solving a linear system of equations that yields a vector of weights determining the linear combination of $\{u(y_i)\}_i$ which acomplishes the approximation. This method only assumes the function to be continuously differentiable up to some order in that neighbourhood, hence its generality.

The fact of the matter is that if one is dealing not with general functions under these conditions but with its subspace of functions satisfying a particular linear partial differential equation, this method can be specialized providing orders of consistency of the approximation at times far superior than that of the original method and always achieving at least the same order. This specialization [1] retains the simplicity of solving a linear system of equations to acquire the approximation required while enhancing its order of consistency.

Keywords: finite difference method, linear partial differential equations, approximation.

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Neural network empowered liquidity pricing in a two-price economy under conic finance settings

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Abstract

In the article at hand neural networks are used to model liquidity in financial markets, under conic finance settings, in two different contexts. That is, on the one hand this paper illustrates how the use of neural networks within a two-price economy allows to obtain accurate pricing and Greeks of financial derivatives, enhancing computational performances compared to classical approaches such as (conic) Monte Carlo. The methodology proposed for this purpose is agnostic of the underlying valuation model, and it easily adapts to all models suitable for pricing in conic financial markets. On the other hand, this article also investigates the possibility of valuing contingent claims under conic assumptions, using local stochastic volatility models, where the local volatility is approximated by means of a (combination of) neural network(s). Moreover, we also show how it is possible to generate hybrid families of distortion functions to better fit the implied liquidity of the market, as well as we introduce a conic version of the SABR model, based on the Wang transform, that still allows for analytical bid and ask pricing formulae.

Keywords: Conic Finance, Liquidity, Neural Network, Concave Distortions.

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tradeoffaucdim: An R package for accessing the trade-off between model performance and dimensionality

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Abstract

In the realm of statistical modeling and machine learning, the pursuit of optimal models requires a delicate equilibrium between predictive performance and the model parsimony and interpretability [1, 2].

This package implements a novel methodology designed to compare the performance of diverse statistical and machine learning techniques on a user-defined data set, explicitly accounting for model dimensionality. This methodology facilitates the identification of models that exhibit high performance but also retain a minimal set of variables, thereby enhancing interpretability and reducing the risk of overfitting. The core of this methodology lies in the iterative fitting of models across a spectrum of dimensionalities, ranging from 1 to k variables. For each model and dimensionality, the methodology assesses the performance as a function of the number of variables used to fit each model. To ensure robust evaluation, the methodology incorporates the computation of confidence intervals based on bootstrapping techniques, along with the application of significance tests to check if model performance differences are statistically significant. This tool also incorporates graphical features that enable visual comparison of model performances.

This tool application to real-life datasets enables researchers to make informed decisions about model and variable selection, choosing a model that strike the right balance between predictive power and ease of interpretation.

Keywords: model building, variable selection, interpretability.

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Formulations and branch-and-cut algorithms for the period travelling salesman problem

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Abstract

Considering a time horizon with different periods, the Period Travelling Salesman Problem (PTSP) is defined on a directed graph with a depot and n customers, in which each customer has a required number of periods to be visited in. The objective of the PTSP is to assign each client to as many periods as it is required while determining the set of routes, one for each period, with minimum cost. Each route starts and ends at the depot and visits the clients assigned to that specific period only once.

The PTSP combines an assignment and a routing problem in a single problem, making it very difficult to solve to optimality, even for instances with few clients. Most of the works in the literature study heuristic methods for the PTSP, and the few exact methods in the literature only consider two periods in the time horizon, which is insufficient to model many real-life routing problems involving periodicity. We used the concept of projection to derive new ILP formulations with fewer variables, in comparison to compact formulations, but exponentially-sized sets of constraints. Furthermore, we propose new valid inequalities for the PTSP in [1]. The conclusions from our theoretical and computational study will be presented for instances with different characteristics.

Keywords: Combinatorial Optimization, Branch-and-cut, Valid inequalities.

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Integer linear programming models for the travelling thief problem

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Abstract

The Traveling Thief Problem (TTP) is a combinatorial optimization problem that combines two well-known \mathcal{NP} -hard problems: the Traveling Salesman Problem (TSP) and the Knapsack Problem (KP). The thief must perform a hamiltonian circuit on a set of nodes, starting and ending at a depot. Items are scattered across the nodes and may be collected as long as the knapsack capacity is not exceeded. Collecting items slows the thief down, and a renting rate is paid for each time unit taken to complete the tour. The objective is to maximize the profit given by the difference between the value of the collected items and the renting expenses. Many heuristic approaches have been introduced in the TTP literature. However, their accuracy is unknown due to their inability to confirm global optima. There is a gap in the TTP literature regarding exact solution approaches, particularly since no integer linear programming (ILP) models have been developed. This study aims to develop ILP models for the TTP and evaluate their computational performance.

Keywords: Traveling thief problem, Integer linear programming, Exact approaches.

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Spline collocation method for a class of singular fractional differential equations of an order $\beta \in (0, 1)$

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Abstract

In the present work, we proposed the collocation method based on splines [3] for the following singular fractional differential equation with non constant coefficients

$$(M^{\beta}D_{0}^{\beta}u)(t) = \sum_{r=1}^{s} b_{r}(t)(M^{\beta_{r}}D_{0}^{\beta_{r}}u)(t) + f(t), \quad 0 < t \le T,$$
(1)

with the following constraints

 $\beta, \beta_r \in \mathbb{R} \quad 0 < \beta < 1, \quad \beta > \beta_r \ge 0, \quad b_r, f \in C^m[0,T], \quad m \in \mathbb{N}_0, \quad 1 \le r \le s.$ (2) The operator M^β is the multiplication operator defined as:

$$(M^{\beta}u)(t) = t^{\beta}u(t) \quad 0 < t \le T \quad u \in C[0,T],$$
(3)

and, D_0^{β} is the fractional differential operator of order β , which is defined as the inverse operator of the Riemann-Liouville integral operator J^{β} for $u \in J^{\beta}C[0,T]$,

$$D_0^{\beta} u = (J^{\beta})^{-1} u$$
 and $(J^{\beta} u)(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-x)^{\beta-1} u(x) dx$

where Γ is the Euler Gamma function. We can rewrite equation (1) into cordial Volterra integral equation by using appropriate transformations, thus by using cordial integral theory [1, 2] we established the uniqueness result for the solution of (1). We formulate the convergence results and proposed numerical results for the illustration of the theoretical established results.

Keywords: Singular fractional differential equation, Cordial Volterra integral operators, Spline collocation method.

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Numerical analysis of shear-dependent unsteady non-Newtonian fluid flow and heat transfer in open-ended cavities

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Abstract

This work aims to elucidate the substantial consequences of non-Newtonian fluid behavior for diverse industrial applications where controlling fluid flow and heat transmission is essential [1]. We seek to acquire a comprehensive understanding of the fluid behavior under various flow configurations [2] through an extensive computational analysis using the finite element method [3] in conjunction with the semi-implicit scheme [4]. Our goal is to investigate the intricate relationships between heat transfer and fluid dynamics in this context. Our research relies on a solid mathematical framework, offering a comprehensive theoretical examination of the properties of non-Newtonian fluids.

Keywords: Finite Element Analysis, Semi-implicit scheme, Heat and mass transfer.

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Global-in-time optimal control of stochastic third-grade fluids

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Abstract

In this talk, we are concerned to understand the velocity tracking control problem for a class of stochastic non-Newtonian fluids. More precisely, we consider stochastic third-grade fluid equation perturbed by infinite-dimensional additive white noise defined on the two-dimensional torus \mathbb{T}^2 . The control acts as distributed random external force. Making use of *infinite-dimensional Ornstein-Uhlenbeck process*, we convert stochastic system into an equivalent pathwise deterministic system which assist us to prove the well-posedness of original stochastic system. Then, we establish the existence and uniqueness of solutions to the corresponding linearized state equation and adjoint equation. Furthermore, we derive an appropriate stability result for the state equation and verify that the Gâteaux derivative of the control-to-state mapping coincides with the solution of the linearized state equation. Finally, we formulate the first-order optimality conditions and prove the existence of an optimal solution.

Keywords: Stochastic third-grade fluids; optimal control; necessary optimality condition, infinite-dimensional Wiener process.

Numerical approximation scheme based on computational method to solve distributed order fractional financial mathematical model

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Abstract

In this study, we present an approximate solution for a distributed order fractional financial mathematical model using a wavelet-based operational matrix approach. First, we construct operational matrices for both distributed order and integer-order fractional derivatives. These matrices are then utilized in conjunction with the standard tau method and collocation points to transform the original problem into a system of linear algebraic equations. Solving this system yields the approximate solution. To assess the robustness and accuracy of the proposed method, we apply it to several test examples and analyze the associated error bounds [1].

Keywords: Distributed order fractional differential equation, Black-Scholes model, Operational matrices, Caputo derivative.

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On the Drazin inverse of double star digraph matrices: Index characterization and pseudo-inverses

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Abstract

A central challenge in spectral graph theory involves characterizing invertibility conditions for graph-associated matrices and deriving explicit expressions for their inverses. This problem naturally extends to the framework of pseudo-invertibility.

We will use the characterization of Drazin invertibility of square matrices by a special nonsingular matrix to derive the index of a matrix associated with a special graph, generalizing results of [2].

Keywords: Drazin inverse, directed graphs, matrix index, pseudo-invertibility.

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Product of formations and Fitting classes on groups and some generalisations

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Abstract

In this talk, we will focus on formations and Fitting classes of groups. A *formation of groups* is a class of groups closed under quotients and subdirect products of finite families, while a *Fitting class of groups* is a class of groups closed under normal subgroups and products of two normal subgroups belonging to the class.

In [1], different definitions of the product of classes of groups have been presented, and studied, particularly regarding the preservation of properties as being a formation or a Fitting class.

Furthermore, research has been conducted with the aim of extending these concepts to congruences and languages on groups, leading to the introduction and study of the concepts of formations and Fitting classes of congruences and languages (see, for example, [2], [3], [4]).

This naturally raises the question of what would be a suitable definition for the product of formations or Fitting classes of congruences and languages. In this talk, we will explore this question for groups and discuss possible generalizations to other algebraic structures, such as Clifford semigroups and inverse semigroups.

Keywords: Formation, Fitting class, idempotent separating congruence, i-normal subsemigroup.

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Continuous data assimilation for a class of third-grade fluids equations in 2d and 3d

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Abstract

With unknown initial data, our aim is to continuously data assimilate for a class of third-grade fluid models [1], in both two- and three-dimensional settings. This is achieved using a literaturebased approach adapted to the finite-dimensional feedback control design for 2*d* incompressible Navier-Stokes equations. The underlying model concerns non-Newtonian fluids, which exhibit complex, non-linear behavior and can be highly sensitive to small variations in initial data and physical parameters. Motivated by these factors, our main objective is to improve the model's accuracy and state predictions for practical applications in fields such as biology, industry and the environment. Through theoretical analysis, this work involves incorporating general interpolant observables to assimilate velocity field observations into the state-governing model, enabling its stabilization via an approximation algorithm [2].

Keywords: Continuous Data Assimilation (CDA), Third-Grade Fluids, PDEs.

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Bilateral base-extension semantics

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Abstract

Bilateralism is the position according to which assertion and denial are perceived as conceptually independent speech acts. This view stands in contrast to the traditional Fregean account, which treats denial of a proposition as merely the assertion of its opposite ([2]). Logical bilateralism demands that systems of logic provide separate conditions for assertion and denial, that are not reducible to each other, often leading to independent definitions of proof rules (for assertion) and refutation rules (for denial). Since it fundamentally concerns the nature of proof and refutation, bilateralism is often explored within proof-theoretic semantics - an approach that aims to elucidate their meaning and the kinds of semantics that arise when they are treated as basic notions. In this work we present a bilateral version of base-extension semantics—one of the most widely studied types of proof-theoretic semantics are shown to be sound and complete with respect to the bilateral dual intuitionistic logic **2Int**. It is also shown that, although independent, the resulting notions of proof and refutation are isomorphic, which allows us to prove semantic duality results (in the sense of [2]) for atomic bases.

Keywords: bi-intuitionistic logic, proof-theoretic semantics, duality.

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A hybrid method for the inverse transmission problem of scattering a time-harmonic acoustic wave by a penetrable object

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Abstract

The inverse transmission problem of acoustic wave scattering by penetrable objects plays a significant role in diverse areas such as radar, sonar, geophysical exploration, medical imaging, and non-destructive testing. We will present a numerical hybrid approach to tackle the inverse acoustic scattering problem for penetrable obstacles using far-field data in two dimensions [1]. This method extends an iterative decomposition framework [2] to address the transmission problem. Initially, the approach reconstructs the scattered and interior fields by applying the far-field equation along with one of the transmission conditions. Subsequently, the second transmission condition is utilized to refine the approximation of the obstacle's boundary through a linearization process. Additionally, we evaluate two linearization techniques: a Newton-type method and a gradient-based method with a penalty term to mitigate high solution oscillations. We will also include a convergence analysis for a related optimization problem [3, 4]. Numerical experiments involving eight incident directions demonstrate the practicality of the method, although it is sensitive to noise.

Keywords: inverse problem, acoustic transmission problem, hybrid method.

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Classes of semigroups and their commuting graphs

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Abstract

The commuting graph of a finite non-commutative semigroup S is a simple graph whose vertices are the non-central elements of S, and where two distinct vertices $x, y \in S \setminus Z(S)$ are adjacent if and only if xy = yx.

In 2011 Araújo, Kinyon and Konieczny [1] showed that for each integer $n \ge 2$ there exists a (finite non-commutative) semigroup whose commuting graph has diameter n. In 2022 Cutolo [3] proved the same result for groups.

In 2016 Bauer and Greenfeld [2] and, independently, Giudici and Kuzma [4] characterized the graphs that arise as commuting graphs of semigroups, which led to the proof that for each integer $n \ge 1$ (respectively, $n \ge 3$) there is a (finite non-commutative) semigroup whose commuting graph has clique number/chromatic number (respectively, girth) equal to n.

In this talk we will see if, by restricting our options to a specific class of semigroups, the possible values for the properties of a commuting graph are still the same. For instance, if we consider the class of completely simple semigroups, is it still true that for each integer $n \ge 3$ there is a completely simple semigroup whose commuting graph has girth equal to n? We will show what the possible values are for some of the properties of a commuting graph when we consider, for example, the classes of completely simple, completely 0-simple and inverse semigroups.

Keywords: Commuting graph, diameter, clique number, chromatic number, girth, inverse semigroup, completely simple semigroup.

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Statistical considerations in variant calling: A performance-based comparative study

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Abstract

Accurate detection of genetic variants is crucial for genomics research and clinical applications, yet it remains challenging due to factors like sequencing errors and complex genomic regions [1]. This study evaluates the performance of various variant calling tools using the well-characterized

NA12878 sample [2]. The methods involved processing high-coverage sequencing data through each variant caller and assessing the results against known benchmarks [6]. Seven widely used variant callers (GATK, FreeBayes, DeepVariant, Samtools, Strelka2, Octopus, Varscan2) were compared by analysis of their precision, recall, and F1-score on chromosome 20 and the whole genome [3].

These tools employ different statistical methodologies. GATK and FreeBayes use Bayesian inference to estimate variant likelihoods, incorporating prior probabilities to improve precision [4]. Samtools relies on genotype likelihoods and probabilistic models, while DeepVariant applies deep learning-based statistical inference to identify variants [5]. Strelka2 enhances variant calling with empirical Bayesian modeling and machine learning, optimizing precision and accuracy [6]. Octopus employs a probabilistic approach with a hidden Markov model for haplotype inference, and Varscan2 utilizes heuristic filtering combined with Fisher's exact test to distinguish somatic from germline mutations. Understanding these statistical approaches is essential for selecting the most appropriate tool based on the characteristics of the data and the research objectives.

DeepVariant demonstrated the highest precision and F1-score for chromosome 20, while Strelka2 excelled in whole-genome sequencing precision [4]. FreeBayes showed higher sensitivity but lower precision, highlighting trade-offs between variant calling tools in terms of accuracy and false positive rates [5]. This study provides insight into the strengths and limitations of each variant caller, guiding researchers and clinicians in selecting appropriate tools for specific needs, and highlights the importance of continuous benchmarking to enhance variant detection accuracy and efficiency. [6].

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Keywords: Variant Calling, Bioinformatics, Statistical Methods, Performance Metrics

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A deep learning approach to parameter estimation in battery modeling for charge-discharge processes

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Abstract

Understanding the behavior of metal anodes in batteries and accurately predicting their performance is a challenge due to the methodological gap between theoretical models and experimental observations. In order to address this challenge, a PDE model describing the voltage profiles behavior of symmetrical coin cells testing the Galvanostatic Discharge-Charge (GDC) protocol has been developed [1, 2] In this talk, based on [3], we propose a hybrid architecture of Convolutional Neural Network and Long-Short Term Memory layers (CNN-LSTM) to estimate some relevant physicochemical parameters in the PDE system that describe GDC cycling of Li/Li symmetric cells. Our results show the neural network ability to capture characteristics of voltage profiles, such as peak and valley, saddle points, and concavity variations [1], that other traditional methods, such as Least Squares (LS) fitting, may overlook. Moreover, our Deep Learning algorithm can successfully estimate parameters also for experimental discharge-charge time series data. These results highlight the robustness of our approach, which allows us to bridge the gap between theory and experiments.

Keywords: Battery PDE modelling, Parameter estimation, Neural Networks.

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Shape optimization problems in PDE, an introduction

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Abstract

Shape optimization problems, though often unnoticed by the naked eye, influence many aspects of our daily lives. Whether it is the minimization of manufacturing costs in the design of water bottles or the strategic packing of luggage in a car trunk, many of our routine decisions involve solving a form of minimization problem. However, as a theoretical mathematician, my focus lies in a different kind of problems, in particular, the ones related to the first eigenvalue of the Dirichlet Laplacian under a measure constraint [1]. In this talk, I will introduce the basics of minimization problems in the context of elliptic PDEs. I will begin with the historic problem that motivated the Isoperimetric

Inequality, which relates the volume of a set to a (d-1)-dimensional measure of its boundary. From this foundation, I will proceed to expose other important results associated to this area, culminating in the Faber-Krahn inequality, a spectral theory result which establishes a relationship between the shape of a domain and the first eigenvalue of the Laplace operator.

Keywords: PDEs, Shape Optimization, Dirichlet Laplacian.

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Proximity measures and outlier detection for mixed-type data

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Abstract

Handling mixed-type datasets, which contain both categorical and numerical variables, presents unique challenges in distance computations. Traditional distances often fail to fully capture the true essence of such data, affecting the performance of clustering and outlier detection techniques.

To address this, numerous distance measures have been proposed, including those by Gower, Huang and Ahmad [1]. My work provides a review of these approaches, aiming to enhance their efficiency and overcome their limitations, while also exploring new ideas and different perspectives. In particular, I explore the potential of Jensen–Shannon Divergence [2], due to its non-linearity properties which offer a distinctive ability to capture complex relationships within the data.

Outlier detection methods play a key role in this topic. They help assess the effectiveness of the proposed distance measures and validating the obtained results, by identifying anomalies in the data that may indicate inconsistencies or underlying patterns that standard distances might fail to capture. For this purpose, I have considered techniques such as KNN, LOF, IForests and the Reference Element, a clustering method adapted to anomaly detection [3].

My research focuses on comparing and developing the previously presented distance measures in controlled datasets, maintaining a critical perspective on strengths and weaknesses and refining their formulations where applicable. Open questions remain regarding the adaptability of these measures across diverse scenarios, inviting further discussion on the challenges and potential directions in mixed-type data analysis.

Keywords: Mixed data, Mixed distances, Outlier detection.

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Maximal operator on variable Lebesgue spaces: Euclidean vs. spaces of homogeneous type

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Abstract

The Hardy–Littlewood maximal function is defined on a quasi-metric measure space (X, d, μ) by

$$Mf(x) = \sup_{B \ni x} \frac{1}{\mu(B)} \int_{B} |f(y)| \, d\mu(y), \quad x \in X,$$

with the supremum taken over all quasi-metric balls $B \subset X$ containing x. The maximal operator $M : f \mapsto Mf$ is a powerful tool in harmonic analysis: the boundedness of M on various function spaces conditions the boundedness of other important operators, like Calderón–Zygmund singular integrals or pseudodifferential operators. We study the maximal operator on variable Lebesgue spaces $L^{p(\cdot)}$.

The boundedness properties of M on the spaces $L^{p(\cdot)}(\mathbb{R}^n)$ —that is, in the Euclidean setting—are well studied. Transferring the main of them to the spaces $L^{p(\cdot)}(X, d, \mu)$ over spaces of homogeneous type (where d is a quasi-metric and μ is a doubling measure) is the subject of my PhD.

In this talk, we present our advances on this way so far: a necessary condition for the boundedness of the maximal operator [1], the self-improvement property [2] and its application in one interpolation result. These new results are compared with their predecessors in \mathbb{R}^n .

Keywords: Hardy-Littlewood maximal operator, variable Lebesgue space, space of homogeneous type.

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Physico-chemical boundary conditions for multiphysics modeling of rechargeable Zn-Air flow batteries

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Abstract

The role of stationary Electrochemical Energy Storage (EES) in the sustainable management of future energy resources is widely recognized. Among various technologies, rechargeable zinc-air flow batteries (RZAFBs) have emerged as a promising solution. [1] Mathematical modeling plays a crucial role in the design and scale-up of RZAFB prototypes by providing insights into their complex multiphysics behavior. [2]

In this work [3], we first reassess and clarify key physico-chemical aspects of a standard multiphysics model for RZAFBs. Subsequently, we develop a comprehensive model based on a system of PDEs that integrates: (i) mass conservation equations for oxygen, zincates, and hydroxide ions; (ii) electrochemical kinetics of both electrodes; (iii) charge conservation; (iv) fluid dynamics for gas and electrolyte phases; and (v) a detailed description of oxygen transfer between the gas and liquid phases. Appropriate boundary conditions (BCs) are imposed for each physical domain, starting from fundamental conditions, such as homogeneous Neumann and Dirichlet BCs, and incorporating problem-specific BCs to accurately capture the system behavior. The developed model is then employed to: (i) predict polarization curves and (ii) map the distribution of the limiting current density.

Overall, the proposed multiphysics model offers a powerful tool for the detailed characterization of RZAFB operation, enabling the identification and mitigation of potential design challenges.

Keywords: Multiphysics modeling, Rechargeable Zinc-Air Flow Battery, Performance characterization.

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The Brown-Halmos theorem for discrete Wiener-Hopf operators

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Abstract

We prove an analogue of the Brown-Halmos theorem for discrete Wiener-Hopf operators acting on separable rearrangement-invariant Banach sequence spaces. This is the first step towards the study of discrete Wiener-Hopf operators or Toeplitz operators in the setting of rearrangement-invariant Banach sequence spaces.

Keywords: Discrete Wiener-Hopf operator; Laurent operator; rearrangement-invariant Banach sequence space; periodic distribution.

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Fredholm criteria for singular integral operators with continuous coefficients on variable Lebesgue spaces

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Abstract

Let $\mathcal{B}_M(\mathbb{R})$ stand for the set of all variable exponents $p(\cdot) : \mathbb{R} \to [1, \infty)$ such that

 $\label{eq:essing} \mathop{\mathrm{ess\,inf}}_{x\in\mathbb{R}} p(x)>1, \qquad \mathop{\mathrm{ess\,sup}}_{x\in\mathbb{R}} p(x)<\infty,$

and the Hardy-Littlewood maximal operator M is bounded on the variable Lebesgue space $L^{p(\cdot)}(\mathbb{R})$. We extend the Fredholm criteria for singular integral operators with continuous coefficients on the Lebesgue space $L^{p}(\mathbb{R}), p \in (1, \infty)$, obtained by Israel Gohberg and Naum Krupnik in the 1970s, to the setting of variable Lebesgue spaces $L^{p(\cdot)}(\mathbb{R})$ with $p(\cdot) \in \mathcal{B}_{M}(\mathbb{R})$.

Keywords: Variable Lebesgue space, Fredholmness, Singular integral operator.

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